Minimizing costs of Covid-19 biomedical waste collection using Mixed Integer Linear Programming approach

Rohan Kalluraya

September 2022

Abstract

In this paper, we will look at a linear programming approach to solve a variant of the multiple traveling salesman problem, with constraints on the working hours of the salesman. First, we will explore the mathematical formulation necessary to solve the problem. Then, we will solve the problem to minimize transport costs while collecting Covid-19 biomedical waste, faced by AG diagnostics - a chain of pathology centres in Pune, India.

1 Introduction

This article aims to minimize transport costs faced by Covid-19 biomedical waste collectors of the AG Diagnostics chain of centers in the city Pune, India. AG diagnostics is a chain of 34 pathology centers for various health checks. Covid-19 biomedical waste is collected once every 48 hours from every center, mandated by the Indian government. There are 2 collectors present at the main center everyday. Each collector must make a tour visiting a group of centers each day and return to the main center at the end of the day.

The task is to find tours for the 2 collectors on each day so that Covid-19 biomedical waste from all centers is collected once every 48 hours and total transport cost is minimized. Each collector must drive for at most 3 hours per day. In order to simplify the problem, we must find 4 tours that cover all centers. Each collector can take one of the tours on one day and another tour on the next day so that all centers are covered over the course of 2 days by the 2 collectors.

The Multiple Traveling Salesman Problem (mTSP) is a generalization of the Traveling Salesman Problem (TSP) in which more than one salesman is allowed. Given a set of centers, one depot where m salesmen are located, and a cost metric, the objective of the mTSP is to determine a tour for each salesman such that the total tour cost is minimized and that each center is visited exactly once by only one salesman.In this paper we solve the mTSP with an added constraint on the driving hours of each salesman.

2 Mathematical Formulation

Here we present an assignment-based integer programming formulation[1], [2] [5]. Consider a graph $G = (V, A)$. Associated with each edge $(i, j) \in A$, is a cost $c_{i,j}$ and a travel time $t_{i,j}$. Let there be n vertices/nodes. We assume that the main center is node 1 and there are 4 collectors at the main center. We define a binary variable $x_{i,j,k}$ for each edge (i, j) such that $x_{i,j,k}$ takes the value 1 if the edge is included in the tour by collector k and 0 otherwise. Another variable $u_{i,k}$ for each node tells the place of the node on a tour by collector k.

2.1 Variables

1. Graph $G = (V, A)$
2. Associated with each edge $(i, j) \in A$, is a cost $c_{i,j}$ and a travel time $t_{i,j}$
3. n vertices/nodes
4. Binary variable $x_{i,j,k}$ for each edge (i, j)
5. Variable $u_{i,k}$ for each node tells the place of the node on a tour by collector k

2.2 Objective

$$\text{Minimize } k \in \{1, 2, 3, 4\} \sum_{(i,j)\in A} c_{i,j} x_{i,j,k} \tag{1}$$

2.3 Constraints

2.3.1 Departures

Ensure that each collector departs from node 1

$$\sum_{j\in V\colon\, (1,j)\in A} x_{1,j,k} = 1, \quad k \in 1, 2, 3, 4 \tag{2}$$

2.3.2 Arrivals

Ensure that each collector returns to node 1

$$\sum_{j\in V\colon\, (j,1)\in A} x_{j,1,k} = 1, \quad k \in 1, 2, 3, 4 \tag{3}$$

2.3.3 One incoming edge

Ensure that exactly one tour enters each node other than the first node.

$$\sum_{i \in V:\,(i,j) \in A} x_{i,j,1} + x_{i,j,2} + x_{i,j,3} + x_{i,j,4} = 1, \quad \forall j \in V, j \neq 1 \tag{4}$$

2.3.4 One outgoing edge

Ensure that exactly one tour exits each node other than the first node.

$$\sum_{j \in V:\,(i,j) \in A} x_{i,j,1} + x_{i,j,2} + x_{i,j,3} + x_{i,j,4} = 1, \quad \forall i \in V, i \neq 1 \tag{5}$$

2.3.5 Flow Conservation

Ensure that the number of collectors entering each node is equal to the number of collectors exiting it.

$$\sum_{j \in V:\,(i,j) \in A} x_{i,j,k} = \sum_{j \in V:\,(i,j) \in A} x_{j,i,k}, \quad \forall i \in V, k \in 1,2,3,4 \tag{6}$$

2.3.6 Miller Tucker Zemlin

Eliminate Subtours

$$n * (1 - x_{i,j,k}) + u_{i,k} \geq u_{j,k} + 1, \quad \forall i \in V, \forall j \in V, i \neq j, j \neq 1, k \in 1,2,3,4 \tag{7}$$

2.3.7 Driving Time

Ensure that each collector drives for less than 3 hours

$$\sum_{i \in V:\,(i,j) \in A} t_{i,j} * x_{i,j,k} \leq 10800, \forall j \in V, k \in 1,2,3,4 \tag{8}$$

3 Results

3.1 Methodology

Using gps coordinates of the centers, I could use the OpenStreetMapX.jl package to find distance and time between every pair of two centers with the following assumptions.

3.2 Speed Assumptions

1. Motorway - 45 kmph
2. Trunk - 30 kmph

3. Primary - 25 kmph
4. Secondary - 25 kmph
5. Tertiary - 20 kmph
6. Residential/Unclassified road - 10 kmph
7. Service road - 10 kmph
8. Living street - 10 kmph

3.3 Cost Assumptions

The cost of renting a truck is Rs. 12 per kilometer.

3.4 Time Constraints

Each truck driver must have less than 3 hours of driving per day.

3.5 Optimizer

The problem is solved using a HiGHS optimizer[4] which is a tool in the JuMP module in Julia. This problem is most suitable for solving linear programs as it is specifically built to do so. By solving the primal and dual it determines the feasibility and solution. Additionally, this is very good with matrices and vectors and does not require much effort to indicate that a variable must be an integer.

3.6 Solution

Primal bound on cost 1786.43687405
Dual bound on cost 1786.25965593
Since duality is zero, the solution is optimal and the minimum cost that will be incurred by the firm is Rs. 1786.44
It required the optimizer 31 minutes to come up with the optimal solution.

4 Conclusion

From the results we can conclude that the problem is feasible and proved that it is optimal. The minimum cost incurred by the firm to collect Covid-19 biomedical waste from all centers will be Rs. 893.22 per day.

This problem is a variant of the multiple traveling salesman problem with a constraint on the driving times of each salesman. We can use this framework to solve larger mTSP with more centers and salesmen.

Scope for future variants of the algorithm include adding constraints to have a maximum time difference between driving routes for all collectors, so that

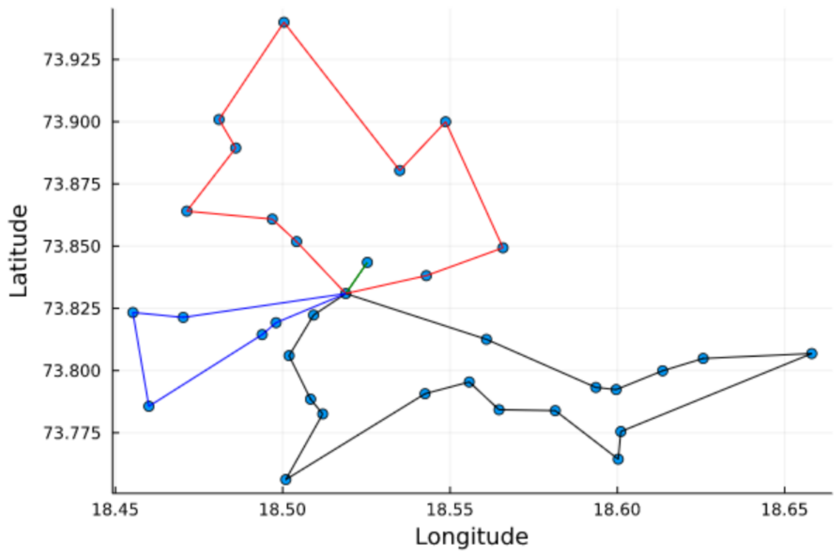

Figure 1: The red, green, black and blue routes display the 4 routes that must be taken by the drivers to minimize costs while driving for at most 3 hours. The nodes represent centers around the city.

fairness of working hours is ensured for each collector. A generalization with multi-depot multiple traveling salesmen[3] can also be studied.

References

[1] Tolga Bektas. "The multiple traveling salesman problem: an overview of formulations and solution procedures". In: *omega* 34.3 (2006), pp. 209–219.

[2] Nicos Christofides, Aristide Mingozzi, and Paolo Toth. "Exact algorithms for the vehicle routing problem, based on spanning tree and shortest path relaxations". In: *Mathematical programming* 20.1 (1981), pp. 255–282.

[3] Moustapha Diaby. "Linear Programming Formulation of the Multi-Depot Multiple Traveling Salesman Problem with Differentiated Travel Costs". In: Nov. 2010. ISBN: 978-953-307-426-9. DOI: 10.5772/12927.

[4] Q. Huangfu and J. A. J. Hall. *Parallelizing the dual revised simplex method - mathematical programming computation.* Dec. 2017. URL: https://link.springer.com/article/10.1007/s12532-017-0130-5.

[5] Gilbert Laporte. "The vehicle routing problem: An overview of exact and approximate algorithms". In: *European journal of operational research* 59.3 (1992), pp. 345–358.